

# Achievable Secrecy Rates over MIMOME Gaussian Channels with GMM Signals in Low-Noise Regime

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**Abstract**—We consider a wiretap multiple-input multiple-output multiple-eavesdropper (MIMOME) channel, where agent Alice aims at transmitting a secret message to agent Bob, while leaking no information on it to an eavesdropper agent Eve. We assume that Alice has more antennas than both Bob and Eve, and that she has only statistical knowledge of the channel towards Eve. We focus on the low-noise regime, and assess the secrecy rates that are achievable when the secret message determines the distribution of a multivariate Gaussian mixture model (GMM) from which a realization is generated and transmitted over the channel. In particular, we show that if Eve has fewer antennas than Bob, secret transmission is always possible at low-noise. Moreover, we show that in the low-noise limit the secrecy capacity of our scheme coincides with its unconstrained capacity, by providing a class of covariance matrices that allow to attain such limit without the need of wiretap coding.

**Index Terms**—multiple-input multiple-output multiple-eavesdropper (MIMOME) Channels, Secrecy Capacity, Physical Layer Security.

## I. INTRODUCTION

The application of information theoretic secrecy principles to widely used communication systems is a rising research topic, in an effort to extend security to the lowest layers. Seminal works from the '70s have established the secrecy capacity of a wiretap channel where agent Alice aims at transmitting a secret message to agent Bob while not revealing any information to an eavesdropper agent Eve [1], [2]. Since then, a number of other scenarios have been investigated, including the broadcast channel with confidential messages (BCC) case where multiple receivers require a common message and possibly a different secret message each [3], the case of secret message transmission over parallel channels [4], [5], fading channels [6] and the multiple-input multiple-output multiple-eavesdropper (MIMOME) case [7]–[9]. In this paper we focus on the MIMOME channel which may find many important applications in wireless communication systems where the transmitter and the receivers are equipped with multiple antennas. In particular, we consider the case in which Alice has perfect knowledge of the channel state information (CSI) of the legitimate channel, whereas she has access only to the statistical description of the channel to Eve. A similar scenario was considered in [10], in which Alice and Bob were assumed to deploy more antennas than Eve, and achievable secrecy rates were obtained by using wiretap coding schemes.

In conventional transmission techniques one codeword in a fixed set (shared among all users) is transmitted by Alice, and the randomness is due to the noise introduced by the channel or by possible fading. Here instead we consider the scenario in which the transmitted codeword is randomly generated from a continuous set, the secret message determining only the statistics of the random codeword through a map that is known to all agents. This approach has been proposed in [11], where a multiplicative Gaussian wiretap channel is considered. In that case the transmitted message is a vector obtained by multiplying a message vector with entries in  $\{0, 1\}$  by a diagonal matrix with independent zero-mean unit-variance Gaussian entries. The message vector is assumed to be sparse (i.e., with many zeros). Assuming that Eve has fewer observations than Bob through a known and fixed channel matrix, it is shown that secrecy transmission is possible and lower and upper bounds to the secrecy capacity are derived, when Alice has knowledge of both channels.

In this paper we consider a wiretap MIMOME channel scenario where Alice has only statistical knowledge on the channel to Eve. Moreover, differently from [11] we do not restrict the transmitted message to be taken from a binary sparse distribution but we allow denser discrete signaling. As a result, the transmitted signal is a Gaussian mixture model (GMM) multivariate vector, whose statistics must be estimated by the receiver. Moreover, we carry out our analysis in the finite-dimension regime, that is, we assume the number of antennas at Alice, Bob and Eve to be finite. We then focus on the low-noise regime, characterizing the achievable rate of this scheme in the absence of noise, where secrecy is provided by the different channels between the agents. We tailor the statistics of the message vector to maximize the secrecy rate. The main results provided by our paper are the following:

- 1) we devise a system for information-theoretic physical layer secrecy where the transmitted signal is generated from one of  $K$  different Gaussian distributions with indices  $\{1, \dots, K\}$  and the informative message is the chosen distribution index;
- 2) we prove that in the low-noise (or equivalently, high-signal-to-noise ratio (SNR)) limit, the secrecy rate that can be achieved by such a system equals the unconstrained capacity,  $\log K$ , even when Alice has only

statistical knowledge of the channel to Eve, and it can be obtained without resorting to wiretap coding techniques;

- 3) we derive the GMM parameters that maximize the secrecy rate achieved by our scheme.

The rest of the paper is organized as follows. Section II describes the system model, providing details on the MIMOME channel, as well as on the specific transmission procedure. Section III focuses on the low-noise regime and we obtain the main results on the achievable rate that can be obtained with the proposed scheme. Numerical results are presented in Section IV, before conclusions are outlined in Section V.

Throughout the paper, vectors (resp., matrices), both deterministic and random, are denoted by boldface lowercase (resp., uppercase) Latin or Greek letters, while  $\log$  denotes the base 2 logarithm.

## II. SYSTEM MODEL

We consider a wireless MIMOME transmission scenario, as depicted in Fig. 1, in which agent Alice aims at transmitting to agent Bob a secret message  $u$  which must be kept secret to a third agent Eve. Alice is equipped with  $n$  antennas, while Bob and Eve have  $m_b$  and  $m_e$  antennas respectively, with  $m_b, m_e < n$ . Between each couple of antennas an additive white Gaussian noise (AWGN) flat static channel is available, whose gain does not change for the duration of the entire transmission. At time  $t$ , Alice transmits a column vector  $\mathbf{x}$  of  $n$  symbols on her antennas, and here we assume that  $\mathbf{x}$  has real entries, leaving the extension to complex-valued transmission for future study. The signal vectors received by Bob and Eve have dimension  $m_b$  and  $m_e$ , respectively, and they can be written as

$$\begin{aligned} \mathbf{y} &= \Phi_b \mathbf{x} + \mathbf{w}_b \\ \mathbf{z} &= \Phi_e \mathbf{x} + \mathbf{w}_e, \end{aligned} \quad (1)$$

where  $\mathbf{w}_b, \mathbf{w}_e \sim \mathcal{N}(\mathbf{0}, I\sigma^2)$  represent AWGN noise. Matrices  $\Phi_b \in \mathbb{R}^{m_b \times n}$  and  $\Phi_e \in \mathbb{R}^{m_e \times n}$ , represent the MIMOME channel. We assume that Alice and Bob know  $\Phi_b$  whereas they have access only to the statistical description of  $\Phi_e$ . Eve is assumed to perfectly know both channel matrices.

We consider an average power constraint on the transmitted signal

$$\mathbb{E} [\mathbf{x}^T \mathbf{x}] \leq P. \quad (2)$$

### A. Transmission Technique

We assume that Alice and Bob agree before transmission on a set of  $K$  column vectors of size  $n$ ,  $\boldsymbol{\mu}_k$ ,  $k = 1, \dots, K$ , and  $K$   $n \times n$  positive semidefinite matrices  $\boldsymbol{\Sigma}_k$ ,  $k = 1, \dots, K$ . These vectors and matrices are assumed also to be known to Eve. Then, at each transmission Alice encodes by an error correcting code the message  $u$  into the message  $c \in \{1, \dots, K\}$  that is sent by generating vector  $\mathbf{x}$  at random, taken from the multivariate normal distribution  $\mathcal{N}(\boldsymbol{\mu}_c, \boldsymbol{\Sigma}_c)$ .

Let  $p_k$  be the probability that message  $k = 1, \dots, K$  is transmitted. Then the input signal  $\mathbf{x} \in \mathbb{R}^n$  follows a GMM

distribution with probability density function (pdf)

$$p_{\mathbf{x}}(\mathbf{a}) = \sum_{k=1}^K p_k \nu(\mathbf{a}; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k), \quad (3)$$

where

$$\nu(\mathbf{a}; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) = \frac{\exp\left(-\frac{1}{2}(\mathbf{a} - \boldsymbol{\mu}_k)^T \boldsymbol{\Sigma}_k^{-1}(\mathbf{a} - \boldsymbol{\mu}_k)\right)}{\sqrt{(2\pi)^n \det \boldsymbol{\Sigma}_k}} \quad (4)$$

is the pdf of a multivariate normal distribution. In other terms, the input signals are drawn with probability  $p_k$  from the Gaussian distribution  $\mathcal{N}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$ . Let the rank of the input covariance matrix associated to the  $k$ -th Gaussian distribution be  $s_k = \text{rank } \boldsymbol{\Sigma}_k$ , and let  $s_{\max} = \max_k s_k$ . The discrete random variable  $c$  corresponding to the choice of the particular Gaussian input class has alphabet  $\{1, \dots, K\}$  and probability mass function (pmf)  $\{p_1, p_2, \dots, p_K\}$ . Note that, in our scenario, the information carried by the transmitted signal is associated with the particular realization of the random variable  $c$ , rather than the actual value of the vector  $\mathbf{x}$ .

Also, the power constraint can be rewritten now as

$$\mathbb{E} [\mathbf{x}^T \mathbf{x}] = \sum_{k=1}^K p_k \text{tr}(\boldsymbol{\Sigma}_k + \boldsymbol{\mu}_k \boldsymbol{\mu}_k^T) \leq P. \quad (5)$$

### B. Problem Statement

Our objective is to determine the GMM parameters, that is  $\{p_k\}$ ,  $\{\boldsymbol{\mu}_k\}$  and  $\{\boldsymbol{\Sigma}_k\}$ , that maximize the achievable secrecy rate. We recall that the supremum on the achievable secrecy rates, i.e., the secrecy capacity, is given in this case by [3]

$$C_s = \max_{(U, c)} [\mathbb{I}(\mathbf{y}; U) - \mathbb{I}(\mathbf{z}; U)], \quad (6)$$

where  $U \rightarrow c \rightarrow (\mathbf{y}, \mathbf{z})$  form a Markov chain and  $\mathbb{I}(a; b)$  is the mutual information between  $a$  and  $b$ . In fact, the secrecy capacity (6) is obtained assuming that all agents have perfect CSI about all channels. When the channel to Eve is known only statistically by Alice, (6) can be achieved with a given outage probability [12]. However, we will present in Section III a signaling strategy that achieves (6) with  $\mathbb{I}(\mathbf{z}; c) \rightarrow 0$  irrespective of the eavesdropper channel realization.

## III. LOW-NOISE REGIME

We focus our analysis on the low-noise regime, i.e., in the limit  $\sigma^2 \rightarrow 0$ , and we consider the achievable secrecy rates that are obtained by imposing  $U = c$  in (6). We assume that the number of antennas at Bob are sufficient to guarantee perfect discrimination among signals coming from the different Gaussian classes in the low-noise regime. Namely, we assume

$$m_b > s_{\max}, \quad (7)$$

and assuming also that the shifted range spaces  $\mathcal{R}(\boldsymbol{\Sigma}_k) + \boldsymbol{\mu}_k$  are all distinct, we have that [13]

$$\lim_{\sigma^2 \rightarrow 0} P_{\text{err}}(\sigma^2) = 0, \quad (8)$$

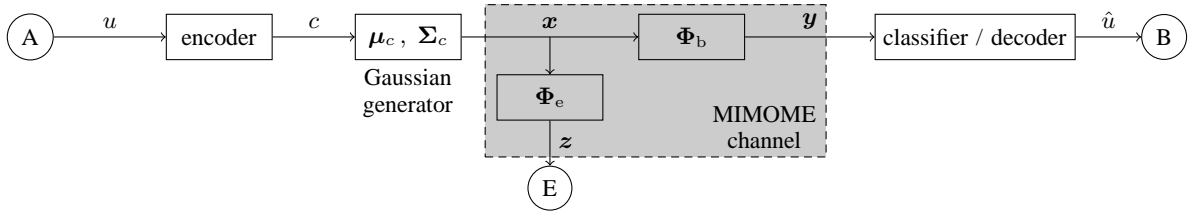


Fig. 1. System model. The confidential message  $u$  is encoded into the information bearing index  $c$ .

where  $P_{\text{err}}$  denotes the misclassification probability associated with the maximum a posteriori (MAP) classifier that estimates the class from which the input signal  $\mathbf{x}$  was drawn from the observation of the measurement vector  $\mathbf{y}$ . Then, by leveraging Fano's inequality [14], we can state that

$$\lim_{\sigma^2 \rightarrow 0} \mathbb{H}(c|\mathbf{y}) = 0, \quad (9)$$

where  $\mathbb{H}(\cdot|\cdot)$  denotes the conditional entropy, and, therefore,

$$\mathbb{H}(\mathbf{y}; c) = \mathbb{H}(c) \leq \log K, \quad (10)$$

where  $\mathbb{H}(c)$  is the entropy of  $c$  and the upper bound in the right hand side is achieved when  $p_k = 1/K$ , for  $k = 1, \dots, K$ .

On the other hand, we assume that the legitimate receiver can leverage an advantage over the eavesdropper in terms of number of antennas, so that  $m_e \leq s_{\text{max}} < m_b$ . Then, consider the information leaked to the eavesdropper,

$$\mathbb{I}(z; c) = h(z) - h(z|c), \quad (11)$$

with  $h(\cdot)$  denoting the differential entropy. Conditioned on  $c = k$ , the random vector  $\mathbf{z}$  follows the Gaussian distribution with mean  $\Phi_e \mu_k$  and covariance  $\Phi_e \Sigma_k \Phi_e^T + \mathbf{I}\sigma^2$  and we can write the conditional differential entropy of  $\mathbf{z}$  given  $c$  as

$$h(z|c) = \sum_{k=1}^K p_k \frac{1}{2} \log [(2\pi e)^{m_e} \det(\Phi_e \Sigma_k \Phi_e^T + \mathbf{I}\sigma^2)]. \quad (12)$$

Moreover, note that also the eavesdropper observation  $\mathbf{z}$  follows a GMM distribution

$$p_{\mathbf{z}}(\mathbf{b}) = \sum_{k=1}^K p_k \nu(\mathbf{b}; \Phi_e \mu_k, \Phi_e \Sigma_k \Phi_e^T + \mathbf{I}\sigma^2). \quad (13)$$

Then, we consider the achievable secrecy rate that is obtained by upper bounding the differential entropy of  $\mathbf{z}$  by that of a multivariate normal distribution with the same mean vector and covariance matrix [14]

$$\begin{aligned} \mu_{\mathbf{z}} &= \mathbb{E}[\mathbf{z}] = \sum_{k=1}^K p_k \Phi_e \mu_k \\ \Sigma_{\mathbf{z}} &= \mathbb{E}[(\mathbf{z} - \mu_{\mathbf{z}})(\mathbf{z} - \mu_{\mathbf{z}})^T] \\ &= \sum_{k=1}^K p_k \Phi_e (\Sigma_k + \mu_k \mu_k^T) \Phi_e^T \\ &\quad - \sum_{k, \ell=1}^K p_k p_{\ell} \Phi_e (\mu_k \mu_{\ell}^T) \Phi_e^T + \mathbf{I}\sigma^2 \end{aligned} \quad (14)$$

respectively, that is by writing

$$h(z) \leq h_G(z) = \frac{1}{2} \log [(2\pi e)^{m_e} \det(\Sigma_{\mathbf{z}})]. \quad (15)$$

Therefore, a lower bound to the secrecy rate achieved in this scenario is

$$R_s(\sigma^2) = \mathbb{I}(\mathbf{y}; c) - h_G(z) + h(z|c) \quad (16)$$

$$\begin{aligned} &= \mathbb{I}(\mathbf{y}; c) - \frac{1}{2} \log [(2\pi e)^{m_e} \det(\Sigma_{\mathbf{z}})] \\ &\quad + \sum_{k=1}^K p_k \frac{1}{2} \log [(2\pi e)^{m_e} \det(\Phi_e \Sigma_k \Phi_e^T + \mathbf{I}\sigma^2)]. \end{aligned} \quad (17)$$

Our aim now is to determine the parameters of the GMM distribution that maximize the low-noise limit of the achievable secrecy rate (17)

$$\begin{aligned} R_s^{\text{LN}} &= \lim_{\sigma^2 \rightarrow 0} R_s(\sigma^2) \\ &= \mathbb{H}(c) - \lim_{\sigma^2 \rightarrow 0} [h_G(z) - h(z|c)]. \end{aligned} \quad (18)$$

Observe that, in the low-noise regime, the class means  $\mu_k$  affect the values of  $R_s^{\text{LN}}$  only through the term  $h_G(z)$ .

The following Lemma states that using zero-mean classes maximizes the low-noise achievable secrecy rate.

*Lemma 1:* Given the positive semidefinite matrix  $\Sigma_{\mathbf{z}}$  in (14), it holds

$$\log \det(\Sigma_{\mathbf{z}}) \geq \log \det \left( \sum_{k=1}^K p_k \Phi_e \Sigma_k \Phi_e^T + \mathbf{I}\sigma^2 \right). \quad (19)$$

*Proof:* Consider the difference matrix

$$\begin{aligned} \Delta &= \Sigma_{\mathbf{z}} - \left( \sum_{k=1}^K p_k \Phi_e \Sigma_k \Phi_e^T + \mathbf{I}\sigma^2 \right) \\ &= \sum_{k=1}^K p_k \Phi_e \mu_k (\Phi_e \mu_k)^T - \sum_{k=1}^K p_k \Phi_e \mu_k \sum_{\ell=1}^K p_{\ell} \mu_{\ell}^T \Phi_e^T. \end{aligned} \quad (20)$$

Note that  $\Delta$  is the covariance matrix of a discrete random vector taking values in the alphabet  $\{\Phi_e \mu_1, \dots, \Phi_e \mu_K\}$  with pmf  $\{p_1, \dots, p_K\}$ , and thus, it is positive semidefinite. Then, by leveraging Weyl's Theorem (see Corollary 4.3.3 in [15]), we can conclude that the ordered eigenvalues of  $\Sigma_{\mathbf{z}}$  are all greater or equal than the corresponding ordered eigenvalues of the matrix on the left hand side of (19), thus proving the inequality. ■

On the basis of Lemma 1, we consider secrecy rates

achieved by zero-mean classes, that is, choosing  $\mu_k = \mathbf{0}$ ,  $k = 1, \dots, K$ . Moreover, on leveraging the convergence properties of differential entropy due to dominated convergence [16], we can write the low-noise achievable secrecy rate with zero-mean classes as

$$R_s^{\text{LN}} = \mathbb{H}(c) - \frac{1}{2} \log \det \left( \sum_{k=1}^K p_k \Phi_e \Sigma_k \Phi_e^T \right) + \frac{1}{2} \sum_{k=1}^K p_k \log \det (\Phi_e \Sigma_k \Phi_e^T), \quad (21)$$

and we can determine the supremum of the secrecy rates achievable in the low-noise regime.

*Theorem 1:* The low-noise secrecy capacity associated with the MIMOME system described in (1) with GMM transmission and discrete input  $c \in \{1, \dots, K\}$  is given by

$$C_s^{\text{LN}} = \lim_{\sigma^2 \rightarrow 0} C_s = \log K. \quad (22)$$

*Proof:* The converse part of the proof is trivial, and it is based on the fact that the secrecy capacity is always lower or equal than the capacity without secrecy constraints, that implies  $C_s \leq \max_c \mathbb{I}(\mathbf{y}; c) \leq \mathbb{H}(c) \leq \log K$ .

In order to provide the achievability part of the proof, we observe that there is a sequence of input distributions that achieves the upper bound  $\log K$  in the asymptotic low-noise regime. The existence of such a sequence is guaranteed, in our scenario, by the fact that, when  $\sigma^2 \rightarrow 0$ , the upper bound in (17) to  $\mathbb{I}(\mathbf{z}; c)$  is a continuous function of the input covariance matrices  $\Sigma_k$ , whereas  $\mathbb{I}(\mathbf{y}; c)$  is not, since (10) holds if the range spaces  $\mathcal{R}(\Sigma_k)$  are all distinct.

We now define a class of covariance matrices, for which we will be able to show that  $\mathbb{I}(\mathbf{z}; c) \rightarrow 0$ . For a given  $\varepsilon \in \mathbb{R}$ , consider the  $K$ -classes, zero-mean, GMM vectors obtained by choosing  $\mu_k = \mathbf{0}$ ,  $p_k = 1/K$  and covariance matrices

$$\Sigma_k(\varepsilon) = \frac{P}{m_b - 1} \mathbf{W}(\varepsilon)^k \begin{bmatrix} \mathbf{I}_{m_b-1} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} [\mathbf{W}(\varepsilon)^k]^T, \quad (23)$$

where  $\mathbf{W}(\varepsilon) \in \mathbb{R}^{n \times n}$  is an orthogonal matrix obtained according to the following Cayley transform [17]:

$$\mathbf{W}(\varepsilon) = (\mathbf{I} - \mathbf{A}(\varepsilon))(\mathbf{I} + \mathbf{A}(\varepsilon))^{-1} \quad (24)$$

from the skew-symmetric matrix

$$\mathbf{A}(\varepsilon) = \begin{bmatrix} 0 & -\varepsilon & \cdots & -\varepsilon \\ \varepsilon & 0 & \cdots & -\varepsilon \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon & \cdots & \varepsilon & 0 \end{bmatrix} \quad (25)$$

Now, it is straightforward to note that

$$\lim_{\varepsilon \rightarrow 0} \mathbf{W}(\varepsilon) = \mathbf{I}, \quad (26)$$

and, therefore, when  $\varepsilon \rightarrow 0$ , all the matrices  $\Sigma_k$  coincide and the two rightmost terms in (18) cancel out. Then, due to the continuity of the log-determinant function of positive definite matrices, we conclude that the supremum of the secrecy rates

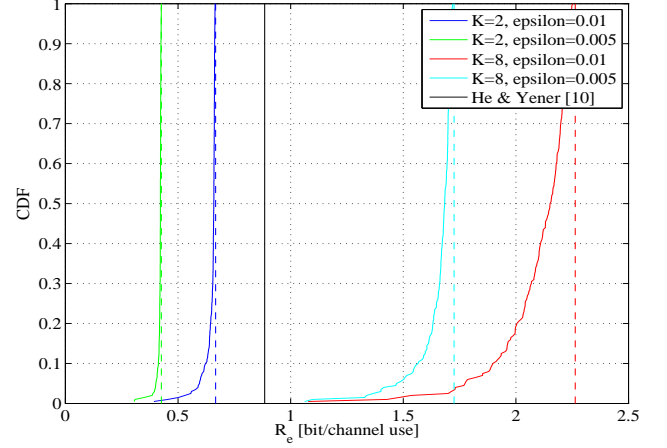


Fig. 2. CDF of the equivocation rates  $R_e$  with fixed  $\Phi_b$ . SNR = 35 dB,  $\varepsilon = 0.01, 0.005$ .  $n = 10$ ,  $m_b = 6$  and  $m_e = 4$ ,  $K = 2, 8$ . The dashed vertical lines represent the transmission rates  $R_c$ .

achieved with this class of covariance matrices, in the low-noise regime is given by

$$\sup_{\varepsilon} R_s^{\text{LN}}(\varepsilon) = \log K. \quad (27)$$

It is relevant to observe that the signaling scheme determined by the input covariance matrices in (23) guarantees that, asymptotically,  $\mathbb{I}(\mathbf{z}; c) \rightarrow 0$  with probability 1 for all the possible realizations of the eavesdropper channel matrix  $\Phi_e$ . This result has been shown to hold in the low-noise limit  $\sigma^2 \rightarrow 0$ , thus implying that, for all values  $\sigma^2 > 0$ , the mutual information to Eve still asymptotically approaches zero, since adding noise decreases the quality of communication. Therefore the proposed scheme provides secrecy even without the knowledge of the eavesdropper channel and, most notably, without requiring the use of wiretap codes. In fact, error correcting coding only can be used and secrecy is obtained directly by leveraging the fact that the mutual information at the eavesdropper can be reduced asymptotically to zero by tuning the parameter  $\varepsilon$ . Moreover, in the low-noise regime, such scheme achieves the secrecy capacity  $\log K$ .

#### IV. NUMERICAL RESULTS

In the previous sections, we have described a transmission strategy which achieves a secrecy rate equal to  $\log K$  in the low-noise regime without the need of wiretap coding. In this section, we focus on finite SNR values, assessing the equivocation rate [18]

$$R_e = [\mathbb{I}(\mathbf{y}; c) - \mathbb{I}(\mathbf{z}; c)]^+. \quad (28)$$

We consider an error correcting code with rate  $R_c = \mathbb{I}(\mathbf{y}; c)$  and recall that, when the equivocation rate  $R_e$  is equal to the transmission rate  $R_c$ , then we have perfect secrecy [1]. We set  $n = 10$ ,  $m_b = 6$  and  $m_e = 4$ , respectively.

Fig. 2 shows the cumulative distribution function (CDF) of the equivocation rates obtained when the legitimate channel

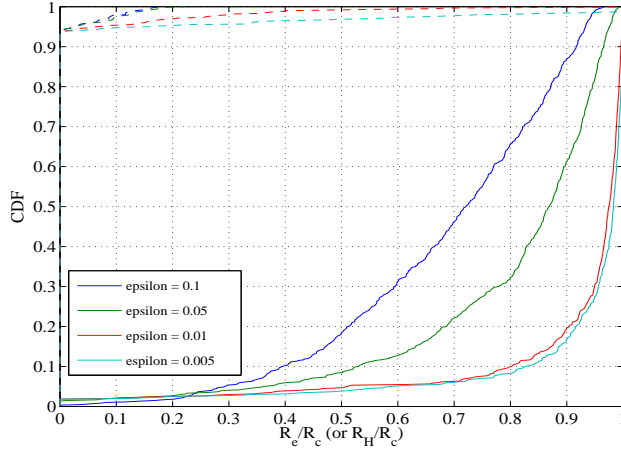


Fig. 3. CDF of the rate  $R_e/R_c$  (solid lines) and of the ratio  $R_H/R_c$  (dashed lines). SNR = 25 dB.  $n = 10$ ,  $m_b = 6$  and  $m_e = 4$ ,  $K = 2$  and  $\varepsilon = 0.1, 0.05, 0.01, 0.005$ .

matrix  $\Phi_b$  contains the first  $m_b$  rows of an  $n$ -dimensional discrete cosine transform (DCT) matrix, whereas the eavesdropper channel matrices are randomly generated with independent identically distributed (i.i.d.) zero-mean, unit-variance, Gaussian entries. The SNR is equal to 35 dB, the numbers of classes of the transmitted signals are  $K = 2$  and  $K = 8$ , and  $\varepsilon = 0.01$  and  $\varepsilon = 0.005$ . We also report the value of the secrecy rate  $R_H$  that is achieved by the wiretap coding scheme described in [10]. We can notice that, when  $K = 2$ , our scheme provides a much lower equivocation rate than the secrecy rate of [10]. Moreover, on increasing the number of transmitted classes to  $K = 8$ , higher equivocation rates are achieved than the secrecy rate of [10], at the expense of a higher information leakage towards Eve.

We then consider the case in which also  $\Phi_b$  is generated at random with i.i.d., zero-mean, unit-variance Gaussian entries. Alice is assumed to know the current realization of the legitimate channel coefficients, and she could arguably optimize the values of  $K$  and  $\varepsilon$  to maximize the equivocation rate under a given constraint on the probability that the leakage to the eavesdropper overcomes a given threshold. Nevertheless, we assess approximately the performance of the system by considering the case in which  $K = 2$ , SNR = 25 dB, and by choosing  $\varepsilon = 0.1, 0.05, 0.01, 0.005$ . Fig. 3 shows the CDF of  $R_e/R_c$ , i.e., the secure fraction of the transmitted rate. For comparison, we also report the CDF of  $R_H/R_c$ , where  $R_c$  is still the code rate of our scheme. We observe that, by taking  $\varepsilon \leq 0.01$ , only the 20% of the channel realizations correspond to information leakages to Eve that are larger than the 10% of the transmitted rate. Moreover, for such values of  $\varepsilon$ , the transmission rate guaranteed by our scheme is higher than the secrecy rate of [10] for the large majority of channel realizations.

## V. CONCLUSIONS

In this paper, we have studied secrecy rates achievable over a MIMOME channel when the transmitted signals are drawn

from a  $K$ -classes GMM distribution, with the information encoded into the index of the chosen Gaussian class. In particular, we have considered the case in which the legitimate user can deploy more antennas than the eavesdropper but they have only statistical knowledge of the channel to Eve, and we have studied the achievable secrecy rates in the low-noise regime.

We have proved that, also when the number of antennas deployed by all the nodes in the network is finite, the low-noise secrecy capacity of this system is given by the unconstrained capacity  $\log K$ . We have also described a class of GMM distributions which achieve the low-noise secrecy capacity by nulling the mutual information at the eavesdropper, thus without the need of wiretap codes.

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